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Supernova constraints on alternative models to dark energy

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ABSTRACT

The recent observations of type Ia supernovae suggest that the universe is accelerating now and decelerated in the recent past. This may be the evidence of the breakdown of the standard Friedmann equation. The Friedmann equation $H^2 \sim \rho$ is modified to be a general form $H^2 = g(\rho)$. Three models with particular form of $g(\rho)$ are considered in detail. The supernova data published by Tonry et al. (2003), Daly & Djorgovski (2003) and Knop et al. (2003) are used to analyze the models. After the best fit parameters are obtained, we then find out the transition redshift $z_{\rm T}$ when the universe switched from the deceleration phase to the acceleration phase.

Key words: cosmological parameters–cosmology: theory–distance scale–supernovae: type Ia–radio galaxies: general

1 INTRODUCTION

The type Ia supernova observations suggest that the expansion of the universe is speeding up rather than slowing down (Perlmuttter et al. 1998,1999; Garnavich et al. 1998; Riess et al. 1998; Tonry et al. 2003; Knop et al. 2003). The measurements of the anisotropy of the cosmic microwave background favor a flat universe (de Bernardis et al. 2000; Hanany et al. 2000; Bennett et al. 2003; Spergel et al. 2003). The observation of type Ia supernova SN 1997ff at $z \sim 1.7$ also supports that the universe was in the deceleration phase in the recent past (Riess 2001). The transition from the deceleration phase to the acceleration phase happened around the redshift $z_{\rm T} \sim 0.5$ (Turner & Riess 2002; Daly & Djorgovski 2003). A form of matter with negative pressure widely referred as dark energy is usually introduced to explain the accelerating expansion. The simplest form of dark energy is the cosmological constant with the equation of state $p_\Lambda = -\rho_\Lambda$. The cosmological constant model can be easily generalized to dynamical cosmological constant models, such as the dark energy model with the equation of state $p_{\rm Q} = \omega_{\rm Q}\rho_{\rm Q}$, where the constant $\omega_{\rm Q}$ satisfies $-1 \leqslant \omega_{\rm Q} < -1/3$. If we remove the null energy restriction $\omega_{\rm Q} \geqslant -1$ to allow supernegative $\omega_{\rm Q} < -1$, then we get the phantom energy models (Alcaniz 2003; Caldwell 2002; Kaplinghat & Bridle 2003). More exotic equation of state is also possible, such as the Chaplygin gas model with the equation of state $p = -A/\rho$ and the generalized Chaplygin gas model with the equation of state $p = -A/\rho^\alpha$ (Kamenshchik, Moschella & Pasquier 2001; Bilic, Tupper & Viollier 2002; Bento, Bertolami & Sen 2002; Carturan & Finelli 2003; Amendola et al. 2003; Cunha, Alcaniz & Lima 2004). In general, a scalar field Q that slowly evolves down its potential $V(Q)$ takes the role of a dynamical cosmological constant (Caldwell, Dave & Steinhardt 1998; Zlatev, Wang & Steinhardt 1999; Ferreira & Joyce 1997,1998; Ratra & Peebles 1988; Perlmutter, Turner & White 1999; Sahni & Starobinsky 2000; Rubano & Barrow 2001; Johri 2002; Di Pietro & Demaret 2001; Ureña-López & Matos 2000; Sen & Sethi 2002; Gong 2002,2004). The scalar field Q is also called the quintessence field. The energy density of the quintessence field must remain very small compared to that of radiation and matter at early epoches and evolves in a way that it started to dominate the universe around the redshift 0.5. There are other forms of dark energy, like the tachyon field (Armendariz-Picon, Damour & Mukhanov 1999; Padmanabhan & Choudhury 2002,2003; Bagla, Jassal & Padmanabhan 2003; Padmanabhan 2003; Padmanabhan & Choudhury 2003).

Although dark energy models are consistent with current observations, the nature of dark energy is still a mystery. Therefore it is possible that the observations show a sign of the breakdown of the standard cosmology. Some alternative models to dark energy models were proposed along this line of reasoning. These models are motivated by brane cosmology (Dvali, Gabadadze & Porrati 2003; Dvali & Turner 2003; Chung & Freese 1999). In this scenario, our universe is a three-brane embedded in a five

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dimensional spacetime. The five dimensional action is

$$S_5 = \frac{-1}{2\kappa_5}\int d^5x\sqrt{G}\,\mathcal{R} + S_{\rm orb} + S_{\rm boundary} + S_{\rm GH},$$

where $\mathcal{R}$ is the Ricci scalar in five dimensions, G is the five dimensional metric determinant, κ_5 is the five dimensional Newton's constant, $S_{\rm orb}$ is the orbifold action, $S_{\rm boundary}$ represents the boundary action and $S_{\rm GH}$ is the Gibbons-Hawking boundary terms. In these models, the usual Friedmann equation $H^2 = 8\pi G\rho/3$ is modified to a general form $H^2 = g(\rho)$ and the universe is composed of the ordinary matter only (Freese & Lewis 2002; Freese 2003; Gondolo & Freese 2003; Sen & Sen 2003a,2003b; Zhu & Fujimoto 2003a,2003b,2003c; Zhu, Fujimoto & He 2003; Wang et al. 2003; Multamaki, Gaztanaga & Manera 2003; Dev, Alcaniz & Jain 2003; Frith 2004; Gong & Duan 2003; Gong, Chen & Duan 2004). One particular example is the brane cosmology with $g(\rho)\sim\rho+\rho^2$. In order to retain the success of the standard cosmology at early times, we require that the modified cosmology recovers the standard cosmology at early times. So $g(\rho)$ must satisfy $g(\rho)\sim\rho$ when $\rho\gg\rho_0$, where ρ_0 is the current matter energy density.

For a spatially flat, isotropic and homogeneous universe with both an ordinary pressureless dust matter and a minimally coupled scalar field Q sources, the Friedmann equations are

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\rm m}+\rho_{\rm Q}), \tag{1}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho_{\rm m}+\rho_{\rm Q}+3p_{\rm Q}), \tag{2}$$

$$\dot{\rho_{\rm Q}} + 3H(\rho_{\rm Q}+p_{\rm Q}) = 0, \tag{3}$$

where dot means derivative with respect to time, $\rho_{\rm m} = \rho_{\rm m0}(a_0/a)^3$ is the matter energy density, a subscript 0 means the value of the variable at present time, $\rho_{\rm Q} = \dot Q^2/2+V(Q)$, $p_{\rm Q} = \dot Q^2/2 - V(Q)$ and $V(Q)$ is the potential of the quintessence field. The modified Friedmann equations for a spatially flat universe are

$$H^2 = H_0^2 g(x), \tag{4}$$

$$\frac{\ddot a}{a} = H_0^2 g(x) - \frac{3H_0^2 x}{2}g'(x)\left(\frac{\rho+p}{\rho}\right), \tag{5}$$

$$\dot\rho + 3H(\rho+p) = 0, \tag{6}$$

where $x = 8\pi G\rho/3H_0^2 = x_0(1+z)^3$ during the matter dominated epoch, $1+z = a_0/a$ is the redshift parameter, $g(x) = x+\cdots$ is a general function of x and $g'(x) = dg(x)/dx$. Note that the universe did not start to accelerate when the other nonlinear terms in $g(x)$ started to dominate. To recover the standard cosmology at early times, we require that $g(x)\approx x$ when $x\gg x_0$. For the matter dominated flat universe, $\rho=\rho_{\rm m}$ and $p = p_{\rm m} = 0$. Let $\Omega_{\rm m0} = 8\pi G\rho_0/3H_0^2$, then $x_0 = \Omega_{\rm m0}$, $g(x_0) = 1$ and $x = \Omega_{\rm m0}(1+z)^3$ during the matter dominated era.

The luminosity distance $d_{\rm L}$ is defined as

$$d_{\rm L}(z) = a_0(1+z)\int_t^{t_0}\frac{dt'}{a(t')}$$

$$= \frac{1+z}{H_0}\int_0^z g^{-1/2}[\Omega_{\rm m0}(1+u)^3]du. \tag{7}$$

The apparent magnitude redshift relation is

$$m(z) = M + 5\log_{10} d_{\rm L}(z) + 25 = \mathcal{M} + 5\log_{10}\mathcal{D}_{\rm L}(z)$$

$$= \mathcal{M} + 5\log_{10}\left\{(1+z)\int_0^z g^{-1/2}[\Omega_{\rm m0}(1+u)^3]du\right\}, \tag{8}$$

where $\mathcal{D}_{\rm L}(z) = H_0 d_{\rm L}(z)$ is the "Hubble-constant-free" luminosity distance, M is the absolute peak magnitude and $\mathcal{M} = M - 5\log_{10}H_0 + 25$. $\mathcal{M}$ can be determined from the low redshift limit at where $\mathcal{D}_{\rm L}(z) = z$. The parameters in our model are determined by minimizing

$$\chi^2 = \sum_i \frac{[m_{obs}(z_i) - m(z_i)]^2}{\sigma_i^2}, \tag{9}$$

where σ_i is the total uncertainty in the observations. The χ^2-minimization procedure is based on MINUIT code. We use the 54 supernova data with both the stretch correction and the host-galaxy extinction correction, i.e., the fit 3 supernova data by Knop et al. (2003), the 20 radio galaxy and 78 supernova data by Daly & Djorgovski (2003), and the supernova data by Tonry et al. (2003) to find the best fit parameters. In the fit, the range of parameter space for $\mathcal{M}$ is $\mathcal{M} = [-3.9,\ 3.2]$, the range of parameter space for $\Omega_{\rm m0}$ is $\Omega_{\rm m0} = [0,\ 4]$.

The transition from deceleration to acceleration happens when the deceleration parameter $q = -\ddot a/aH^2 = 0$. From equations (4) and (5), we have

$$g[\Omega_{\rm m0}(1+z_{\rm T})^3] = \frac{3}{2}\Omega_{\rm m0}(1+z_{\rm T})^3 g'[\Omega_{\rm m0}(1+z_{\rm T})^3], \tag{10}$$

$$q_0 = \frac{3}{2}\Omega_{\rm m0}g'(\Omega_{\rm m0}) - 1. \tag{11}$$

To compare the modified model with the dark energy model, we make the following identification

$$\omega_Q = \frac{xg'(x)-g(x)}{g(x)-x}. \tag{12}$$

2 CHAPLYGIN GAS MODEL

The chaplygin gas model $p = -A/\rho^\alpha$ in the framework of alternative model to dark energy is

$$g(x) = x + \Omega_{\rm Q0}[A_s + (1-A_s)(x/\Omega_{\rm m0})^\beta]^{1/\beta},$$

where $\Omega_{\rm Q0} = 1-\Omega_{\rm m0}$, $\beta = 1+\alpha$ and $A_s = (8\pi G/3H_0^2\Omega_{\rm Q0})^\beta A$. The $\alpha = 1$ model is motivated by a d-brane in $d+2$ spacetime. Since $g'(x) = 1+\Omega_{\rm Q0}(1-A_s)[A_s+(1-A_s)(x/\Omega_{\rm m0})^\beta]^{1/\beta-1}(x/\Omega_{\rm m0})^\beta$, so

$$\frac{\Omega_{\rm m0}}{2\Omega_{\rm Q0}}(1+z_{q=0})^3[A_s+(1-A_s)(1+z_{q=0})^{3\beta}]^{1-1/\beta}$$

$$= A_s - \frac{1}{2}(1-A_s)(1+z_{q=0})^{3\beta}, \tag{13}$$

$$q_0 = \frac{1}{2} - \frac{3}{2}A_s(1-\Omega_{\rm m0}), \tag{14}$$

$q_0<0$ gives that $A_s > (1-\Omega_{\rm m0})^{-1}/3$. To retain the success of the standard model at early epoches, we require $g(x)\approx x$ when $x\gg 1$. In other words, we require $A_s\sim 1$. Therefore, we have the following constraints

$$(1-\Omega_{\rm m0})^{-1}/3 < A_s < 1, \tag{15}$$

$$A_s \sim 1. \tag{16}$$

The best fits to the 54 supernovae by Knop et al. (2003) are $\Omega_{\rm m0} = [0,\ 0.44]$ centered at almost zero, $A_s = [0.99,\ 1]$

centered at almost one and $\beta = [1,\ 32.3]$ centered at 22.0 with $\chi^2 = 43.9$. The best fits to the 98 radio galaxy and supernova data compiled by Daly & Djorgovski (2003) are $\Omega_{\rm m0} = [0,\ 0.27]$ centered at 0.26, $A_s = [0.65,\ 1]$ centered at 0.97 and $\beta = 1$ with $\chi^2 = 87.8$. The best fits to the 172 supernovae with redshift $z > 0.01$ and $A_v < 0.5$ mag (Tonry et al. 2003) are $\Omega_{\rm m0} = [0,\ 0.3]$ centered at 0.1, $A_s = [0.99,\ 1]$ centered at almost one and $\beta = [1.5,\ 23.7]$ centered at 16.1 with $\chi^2 = 169.5$. The best fits to the 194 supernovae by Tonry et al. (2003) and Barris et al. (2004) are $\Omega_{\rm m0} = [0,\ 0.36]$ centered at 0.19, $A_s = [0.99,\ 1]$ centered at almost one and $\beta = [1.3,\ 25.2]$ centered at 14.9 with $\chi^2 = 195.1$. The best fits to all the data combined are $\Omega_{\rm m0} = [0,\ 0.34]$ centered at 0.21, $A_s = [0.99,\ 1]$ centered at almost one and $\beta = [1.0,\ 21.2]$ centered at 13.7 with $\chi^2 = 331.3$. From the above results, we conclude that the generalized Chaplygin gas model tends to be the Λ model.

3 GENERALIZED CARDASSIAN MODEL

The model is

$$g(x) = x[1 + Bx^{\alpha(n-1)}]^{1/\alpha},$$

where $B = (\Omega_{\rm m0}^{-\alpha} - 1)/\Omega_{\rm m0}^{\alpha(n-1)}$, $\alpha > 0$ and $n < 1 - 1/3(1 - \Omega_{\rm m0}^{\alpha})$. When $n = 0$, $g(x) = B^{1/\alpha}(1 + x^\alpha/B)^{1/\alpha}$ which is the case studied by Freese (2003). For the special case $\alpha = 1$ and $n = 0$, $g(x) = x + B$ which is the standard cosmology with a cosmological constant. From a purely phenomenological point of view we may think that gravity is modified in such a way that acceleration kicks in when the energy density approaches a certain value (Carroll 2003). The model is motivated from a three-brane located at the Z_2 symmetry fixed plane of a five dimensional spacetime. Chung and Freese showed that if one parametrizes the Hubble rate in terms of the brane energy density, then Cardassian model is derived with suitable choice of the five dimensional energy momentum tensor (Chung & Freese 1999). The generalized Cardassian model gives

$$\begin{aligned} g'(x) &= [1 + Bx^{\alpha(n-1)}]^{1/\alpha} \\ &\quad + (n-1)Bx^{\alpha(n-1)}[1 + Bx^{\alpha(n-1)}]^{1/\alpha - 1}, \end{aligned} \tag{17}$$

Combining equation (17) with equations (10) and (11), we get

$$1 + z_{\rm T} = [(\Omega_{\rm m0}^{-\alpha} - 1)(2 - 3n)]^{1/3\alpha(1-n)}, \tag{18}$$

$$q_0 = \frac{1}{2} + \frac{3}{2}(n-1)(1 - \Omega_{\rm m0}^{\alpha}). \tag{19}$$

If we think the generalized Cardassian model as ordinary Freidmann universe composed of matter and dark energy, we can identify the following relationship for the parameters in the Cardassian and quintessence models

$$\omega_{\rm Q0} = \frac{(n-1)(1 - \Omega_{\rm m0}^{\alpha})}{1 - \Omega_{\rm m0}}.$$

There are four parameters in the fits: the mass density $\Omega_{\rm m0}$, the parameters n and α, as well as the nuisance parameter $\mathcal{M}$. The range of parameter space explored is: $n = [-10,\ 0.66]$ and $\alpha = (0,\ 10^4]$. The best fits to the supernova data (Daly & Djorgovski 2003; Tonry et al. 2003; Knop et al. 2003) generally give very large $\alpha > 100$, so $B \approx \Omega_{\rm m0}^{-\alpha n}$ and the transition redshift is weakly dependent

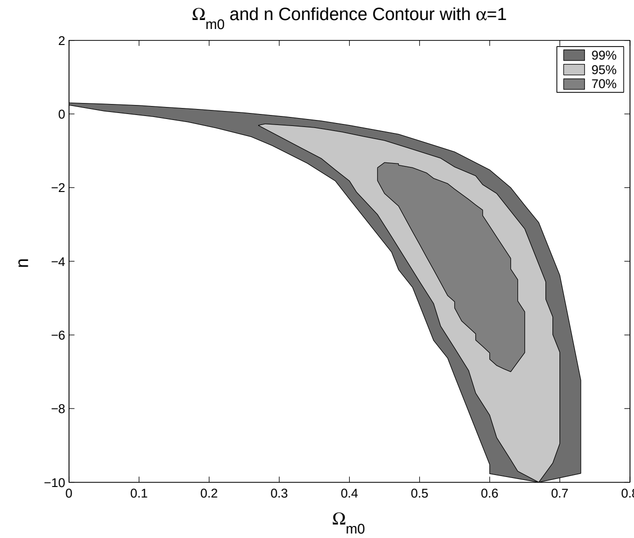

Figure 1. The 70%, 95% and 99% confidence contours of $\Omega_{\rm m0}$ and n in Cardassian model from the 54 supernovae given by Knop et al. (2003)

on α. Furthermore, χ^2 changes very little when α changes over a fairly large range. In other words, the generalized Cardassian model differs little from the Cardassian model with $\alpha = 1$. So we will discuss the Cardassian model in more detail below.

3.1 Cardassian Model

The Cardassian model is the special case $\alpha = 1$ of the generalized Cardassian model. This model is equivalent to the dark energy model with a constant equation of state $p_{\rm Q} = \omega_{\rm Q}\rho_{\rm Q}$ in the sense of dynamical evolution. The equivalence is provided by $n = 1 + \omega_{\rm Q0}$ and the equivalent dark energy potential is $V(Q) = A[\sinh k(Q/\alpha + C)]^{-\alpha}$ with $\alpha = -2 - 2/(n-1)$. The best fits to the 54 supernovae by Knop et al. (2003) are $\Omega_{\rm m0} = 0.56^{+0.09}_{-0.12}$, $n = -3.6^{+2.2}_{-3.4}$ and $\chi^2 = 43.73$. The $\Omega_{\rm m0}$ and n contour plot is shown in figure 1.

The best fits to the 98 radio galaxy and supernova data compiled by Daly & Djorgovski (2003) are $\Omega_{\rm m0} = 0.14^{+0.32}_{-0.14}$, $n = 0.26^{+0.21}_{-0.91}$ and $\chi^2 = 87.45$. The contour plot is shown in figure 2.

The best fits to the 172 supernovae with redshift $z > 0.01$ and $A_v < 0.5$ mag (Tonry et al. 2003) are $\Omega_{\rm m0} = 0.48^{+0.09}_{-0.18}$, $n = -1.2^{+1.1}_{-1.9}$ and $\chi^2 = 171.4$. The best fits to the 194 supernovae by Tonry et al. (2003) and Barris et al. (2004) are $\Omega_{\rm m0} = 0.51^{+0.08}_{-0.16}$, $n = -1.2^{+1.1}_{-1.9}$ and $\chi^2 = 196.7$. The contour plot is shown is figure 3. The plot agrees well with the figure 13 in Tonry et al. (2003) and the figure 6 in Frith (2004).

The above results are summarized in table 1. Combining the above results, we find that $\Omega_{\rm m0} = [0,\ 0.61]$ centered at 0.45 and $n = [-3.1,\ 0.5]$ centered at -0.6 at 99% confidence level. The 99% contour plot is shown in figure 4. Take $\Omega_{\rm m0} = 0.3$, we get $z_{\rm T} = 0.35$, $q_0 = -3.1$ and $\omega_{\rm Q0} = -3.44$ when $n = -2.44$; $z_{\rm T} = 0.57$, $q_0 = -0.24$ and $\omega_{\rm Q0} = -0.7$ when $n = 0.3$. These results are consistent with those obtained

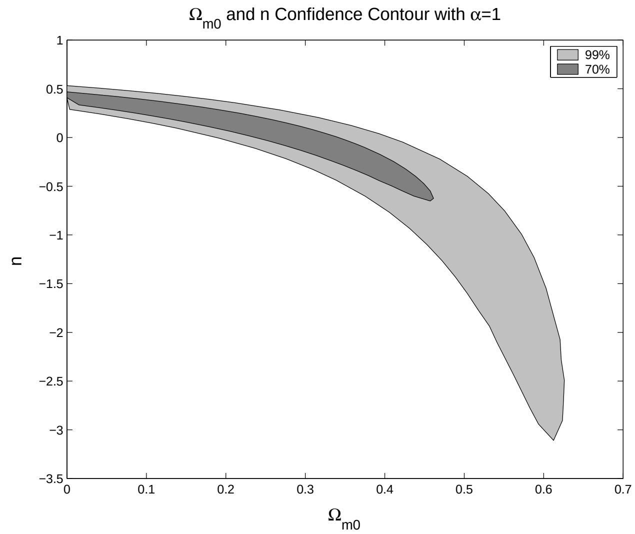

Figure 2. The 70% and 99% confidence contours of $\Omega_{\rm m0}$ and n in Cardassian model from 20 radio galaxies and 78 supernovae compiled by Daly & Djorgovski (2003)

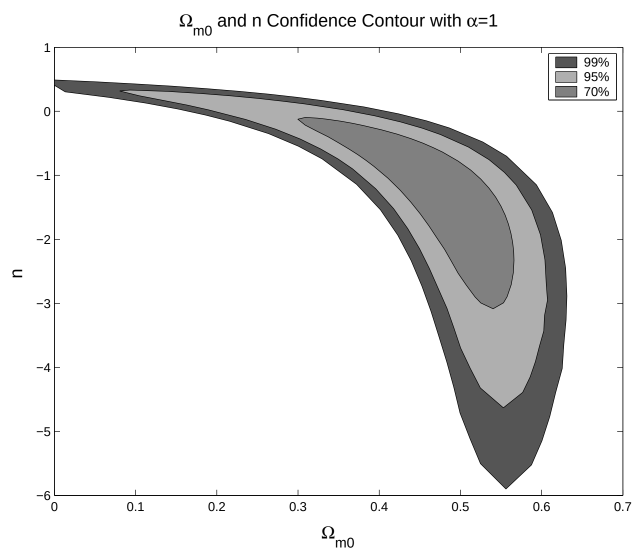

Figure 3. The 70%, 95% and 99% confidence contours of $\Omega_{\rm m0}$ and n in Cardassian model from the 172 supernovae with $z > 0.01$ and $A_v < 0.5$ mag listed in Tonry et al. (2003)

by Zhu & Fujimoto (2003a,b,c), Zhu, Fujimoto & He (2003) and Sen & Sen (2003b).

4 MODEL 3

The last model we would like to consider is $g(x) = [a+\sqrt{a^2+x}]^2$ (Deffayet et al. 2002; Dvali & Turner 2003), where $a = (1-\Omega_{\rm m0})/2$. This model arises from the brane world theory by Dvali, Gabadadze & Porrati (2003) in which gravity appears four dimensional at short distances while modified at large distances. For this model, we find that the equivalent dark energy equation of state parameter $\omega_{\rm Q0}$, q_0 and the transition redshift $z_{\rm T}$ from decelerated

Table 1. Best fits to Cardassian model

Fit #	$\Omega_{\rm m0}$ 70%	$\Omega_{\rm m0}$ 99%	n 70%	n 99%	χ^2
1	$0.56^{+0.09}_{-0.12}$	$0.56^{+0.17}_{-0.56}$	$-3.6^{+2.2}_{-3.4}$	$-3.6^{+3.9}_{-6.6}$	43.73
2	$0.14^{+0.32}_{-0.14}$	$0.14^{+0.48}_{-0.14}$	$0.26^{+0.21}_{-0.91}$	$0.26^{+0.27}_{-3.37}$	87.45
3	$0.48^{+0.09}_{-0.18}$	$0.48^{+0.15}_{-0.48}$	$-1.2^{+1.1}_{-1.9}$	$-1.2^{+1.7}_{-4.7}$	171.4
4	$0.51^{+0.08}_{-0.16}$	$0.51^{+0.14}_{-0.51}$	$-1.2^{+1.1}_{-1.9}$	$-1.2^{+1.8}_{-4.9}$	196.7
5	$0.45^{+0.10}_{-0.19}$	$0.45^{+0.16}_{-0.45}$	$-0.6^{+0.7}_{-1.1}$	$-0.6^{+1.1}_{-2.5}$	332.0

Fit 1 is the fit to the 54 supernova data from Knop et al. (2003), fit 2 is the fit to the 98 data points from Daly & Djorgovski (2003), fit 3 is the fit to the 172 supernova data from Tonry et al. (2003), fit 4 is the fit to the 194 supernova data from Tonry et al. (2003) and Barris et al. (2004) and fit 5 is the fit to the above data combined.

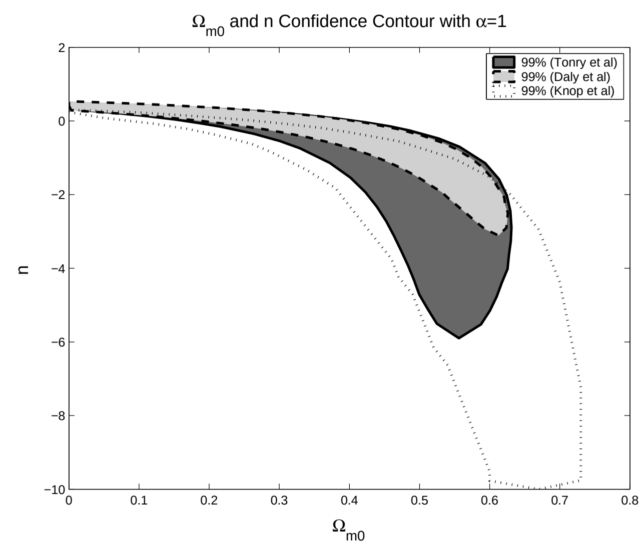

Figure 4. The 99% confidence contours of $\Omega_{\rm m0}$ and n in Cardassian model from the sample data in Tonry et al. (2003), Daly & Djorgovski (2003) and Knop et al. (2003)

expansion to accelerated expansion are

$$\omega_{\rm Q0} = -1/(1+\Omega_{\rm m0}), \tag{20}$$

$$q_0 = \frac{2\Omega_{\rm m0}-1}{1+\Omega_{\rm m0}}, \tag{21}$$

$$1+z_{\rm T} = \left[\frac{2(1-\Omega_{\rm m0})^2}{\Omega_{\rm m0}}\right]^{1/3}. \tag{22}$$

Applying the 54 supernova data with host-galaxy extinction correction (Knop et al. 2003), we find that $\Omega_{\rm m0} = 0.19^{+0.07}_{-0.05}$ and $\chi^2 = 45.71$. The 20 radio galaxy and the 78 supernova data (Daly & Djorgovski 2003) give the best fit $\Omega_{\rm m0} = 0.18 \pm 0.03$ and $\chi^2 = 87.6$. The best fit from the 172 supernovae with redshift $z > 0.01$ and $A_v < 0.5$ mag is $\Omega_{\rm m0} = 0.17^{+0.04}_{-0.03}$ and $\chi^2 = 175.2$. If we use the 194 supernovae given by Tonry et al. (2003) and Barris et al. (2004), we find that $\Omega_{\rm m0} = 0.22^{+0.04}_{-0.03}$ and $\chi^2 = 200.6$. The above results are summarized in table 2. Combining the above results, we get $\Omega_{\rm m0} = 0.20 \pm 0.02$ at 1σ level or

Table 2. Best fits to model 3

Data source	#	Ω_{m0} 1σ	Ω_{m0} 3σ	χ^2	z_T	ω_{Q0}
Knop	54	$0.19^{+0.07}_{-0.05}$	$0.19^{+0.2}_{-0.12}$	45.71	0.90	-0.84
Daly	98	0.18 ± 0.03	$0.18^{+0.1}_{-0.07}$	87.6	0.95	-0.85
Tonry	172	$0.17^{+0.04}_{-0.03}$	$0.17^{+0.14}_{-0.09}$	175.2	1.0	-0.85
Barris	194	$0.22^{+0.04}_{-0.03}$	$0.22^{+0.13}_{-0.09}$	200.6	0.77	-0.82
Combined	346	0.20 ± 0.02	$0.20^{+0.07}_{-0.06}$	334.6	0.86	-0.83

$\Omega_{m0}=0.20^{+0.07}_{-.0.06}$ at 3σ level. If we take $\Omega_{m0}=0.15$, then we have $q_0=-0.61$ and $z_T=1.13$. If we take $\Omega_{m0}=0.21$, then we have $q_0=-0.48$ and $z_T=0.81$. If we take $\Omega_{m0}=0.28$, then we have $q_0=-0.34$ and $z_T=0.55$. These results are consistent with those obtained by Deffayet et al. (2002).

5 DISCUSSIONS AND CONCLUSIONS

A general function $g(x)$ of the ordinary matter density was used to explain the current accelerating expansion of the universe. In this model, no exotic matter form is needed. This approach is equivalent to dark energy model building approach in the sense of dynamical evolution of the universe because we can map the modified part of energy density to dark energy. The function $g(x)$ satisfies the following conditions: (1) $g(x_0)=1$; (2) $g(x)\approx x$ when $z\gg 1$; (3) $g(x_0)>3x_0g'(x_0)/2$, where $x=\Omega_{m0}(1+z)^3+\Omega_{r0}(1+z)^4$. The generalized Chaplygin gas model tends to take $A_s=1$ which becomes the dark energy model with a cosmological constant. Therefore the generalized Chaplygin gas model is disfavored in the framework of alternative models although it is a viable dark energy model. Unlike the gravitational lensing constraint (Dev, Alcaniz & Jain 2003), the supernova data do not provide tight constraint on the generalized Cardassian model. A fairly large range of parameters from the generalized Cardassian model are consistent with the supernova data. For the Cardassian model, the supernova data give $\Omega_{m0}=[0,\ 0.62]$ and $n=[-3.11,\ 0.3]$ at the 99% confidence level. If we have better constraint on the transition redshift z_T, then we will be able to distinguish the Cardassian model from the Λ model because the Λ model is the special case $n=0$. For the model 3, we find that $\Omega_{m0}=[0.12,\ 0.28]$ with 99.7% confidence. Only the upper limit gives $z_T\sim 0.5$.

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